

Worked Solutions

Edexcel C4 Paper C

1. (a) $\frac{5x+7}{(x+1)(x+2)} = \frac{2}{x+1} + \frac{3}{x+2}$ (3)

$$y = 2(x+1)^{-1} + 3(x+2)^{-1}$$

$$\frac{dy}{dx} = -2(x+1)^{-2} - 3(x+2)^{-2}$$

$$\frac{d^2y}{dx^2} = 4(x+1)^{-3} + 6(x+2)^{-3}$$

when $x = 1$, $\frac{d^2y}{dx^2} = \frac{4}{2^3} + \frac{6}{3^3} = \frac{13}{18}$ (3)

2. $V = 36h^2 \Rightarrow \frac{dV}{dh} = 72h$

we are given $\frac{dV}{dt} = 24$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

when $h = 2$, $24 = 72 \times 2 \times \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{24}{144} = \frac{1}{6} \text{ cm s}^{-1}$ (5)

3. (a) $(1+2x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(2x)^2$
 $+ \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3 \cdot 2}(2x)^3 + \dots$
 $= 1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3$ (4)

(b) valid for $-\frac{1}{2} < x < \frac{1}{2}$

(c) $(1+ax)(1-x + \frac{3}{2}x^2 - \frac{5}{2}x^3) = 1 - x + \frac{3}{2}x^2 - \dots$

$$-1 + a = 3, \quad \therefore a = 4$$

$$\text{coef. of } x^3 = -\frac{5}{2} + \frac{3 \cdot 4}{2} = \frac{7}{2}$$

4. (a) when $\theta = \frac{\pi}{2}$, $x = \pi + 1$ and $y = 0$

$$\frac{dy}{d\theta} = -\sin\theta, \quad \frac{dx}{d\theta} = 2 + \cos\theta$$

$$\frac{dy}{dx} = \frac{-\sin\theta}{2 + \cos\theta}$$

when $\theta = \frac{\pi}{2}$, gradient of tangent = $-\frac{1}{2}$

equation of tangent is $y - 0 = -\frac{1}{2}(x - \pi - 1)$

$$2y = -x + \pi + 1$$

$$2y + x = \pi + 1$$

(b) At stationary points $\frac{dy}{dx} = 0$, $\therefore \sin\theta = 0$

$$\theta = 0, \pi, 2\pi$$

$$\theta = 0, \quad x = 0, \quad y = 1$$

$$\theta = \pi, \quad x = 2\pi, \quad y = -1$$

$$\theta = 2\pi, \quad x = 4\pi, \quad y = 1$$

There are stationary points at $(0, 1)$, $(2\pi, -1)$, $(4\pi, 1)$

5. (i) x 0 1 2 3
 $\ln(1 + \sin x)$ 0 0.61056 0.64674 0.13201

$$I = \frac{1}{2} [0 + 0.13201 + 2(0.61056 + 0.64674)]$$

$$= 1.32 \text{ to 3 sig. fig.} \quad (5)$$

(ii) $\int_0^3 \ln(1 + \sin x)^5 dx = \int_0^3 5 \ln(1 + \sin x) dx$

i.e. $5 \times 1.32 = 6.6$ (2)

6. (a) $t = 0, N = 5000$

$$5000 = Ae^0$$

$$A = 5000 \quad (2)$$

(b) $t = 4, N = 4000 \Rightarrow 4000 = 5000 e^{-4k}$

$$e^{4k} = \frac{5}{4}$$

$$4k \ln e = \ln \frac{5}{4}$$

$$k = \frac{1}{4} \ln \frac{5}{4} \quad (0.055786\dots) \quad (4)$$

(c) when $t = 8, N = 5000 e^{-\left(\frac{1}{4} \ln \frac{5}{4}\right) \times 8} = 3200$ (3)

7. (a) $(x-3)(x-1)$

$$\text{let } \frac{2x}{(x-3)(x-1)} \equiv \frac{A}{x-3} + \frac{B}{x-1}$$

$$= \frac{3}{x-3} - \frac{1}{x-1} \quad (e)$$

(b) $\frac{1}{y} dy = \int \frac{3}{x-3} - \frac{1}{x-1} dx$

$$\ln y = 3 \ln(x-3) - \ln(x-1) + c$$

$$y = \frac{1}{3}, x = 4: \ln \frac{1}{3} = 3 \ln 1 - \ln 3 + c$$

$$c = \ln \frac{1}{3} + \ln 3 = 0$$

$$\text{so } \ln y = \ln \frac{(x-3)^3}{(x-1)} \quad y = \frac{(x-3)^3}{(x-1)}$$

8. (a) direction of l is $\begin{pmatrix} 3 \\ -6 \\ -9 \end{pmatrix}$

$$\text{equation of } l \text{ is } r = \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -6 \\ -9 \end{pmatrix}.$$

(b) at point of intersection $1 + 3\lambda = 4 + \mu$

$$6 - 6\lambda = 8 + 2\mu$$

$$1 - 9\lambda = -4 - \mu$$

$$\text{solving we obtain } \lambda = \frac{1}{3}, \mu = -2$$

$$\text{point of intersection is } (2, 4, -2)$$

$$(c) \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ k \\ 5 \end{pmatrix} = \sqrt{6} \sqrt{25 + k^2 + 25} \cos 60^\circ$$

$$5 + 2k - 5 = \sqrt{6} \sqrt{50 + k^2} \times \frac{1}{2}$$

$$(4k)^2 = 6(50 + k^2)$$

$$16k^2 = 300 + 6k^2$$

$$10k^2 = 300$$

$$k^2 = 30$$

$$k = \sqrt{30}$$

(3)

9. (a) $\left(\frac{1}{4}, 0\right)$

(1)

$$(b) \frac{dy}{dx} = 2x \cdot \frac{1}{2}(1-4x)^{-\frac{1}{2}}(-4) + 2\sqrt{1-4x}$$

$$= \frac{-4x}{\sqrt{1-4x}} + 2\sqrt{1-4x} = \frac{-4x + 2(1-4x)}{\sqrt{1-4x}} = \frac{2-12x}{\sqrt{1-4x}}$$

$$\frac{dy}{dx} = 0 \text{ at } x = \frac{1}{6} \text{ and at that point}$$

$$(c) \text{ area} = \int_0^{\frac{1}{4}} 2x\sqrt{1-4x} \, dx$$

$$= -\frac{1}{4} \int_1^0 \left(\frac{1-u}{2}\right) u^{\frac{1}{2}} \, du$$

$$= \frac{1}{8} \int_0^1 \left(u^{\frac{1}{2}} - u^{\frac{3}{2}}\right) \, du$$

$$= \frac{1}{8} \left[\frac{2}{3} u^{\frac{3}{2}} - \frac{2}{5} u^{\frac{5}{2}} \right]_0^1$$

$$= \frac{1}{8} \left[\frac{2}{3} - \frac{2}{5} - 0 \right] = \frac{1}{30}$$